

# Optimal ways of color mixing for high-quality white-light LED sources

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Optimization of colour mixing in white-light sources based on light-emitting diodes (LEDs) only and those utilizing partial light conversion by a yellow YAG:Ce<sup>3+</sup> phosphor is carried out by simulations. The trade-off between the colour rendering index and efficacy of the white-light sources is examined, accounting for the efficiency of electricity-to-light conversion in state-of-the-art LEDs. The asymmetry and widths of the LED emission spectra, as well as the LED efficiencies, are regarded to depend sub-

stantially on the peak emission wavelength. The optimization has provided the optimal peak emission wavelengths of individual LEDs and the power fraction of their contribution to the total emission spectra, allowing estimations of the maximum achievable values of the colour rendering index and efficacy for the white-light sources considered. Their comparison enables choosing the most advantageous solutions for producing high-quality white light.

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**1 Introduction** White-light sources suitable for solid-state lighting are required to provide good colour rendition usually estimated in terms of colour rendering index (CRI), on the one hand, and high efficacy dependent on both the total (integral) emission spectrum of the light source and its efficiency of electricity-to-light conversion, on the other hand. Simultaneous achievement of high CRI and efficacy is rather problematic because of the well-known trade-off between these parameters. Therefore, finding a compromise in the choice of CRI and efficacy suitable for particular lighting applications is necessary by applying one or another criterion.

Normally luminous efficacy of radiation (LER) is considered instead of the actual efficacy of a light source in order to simplify the optimization of colour mixing. Earlier theoretical study [1] showed that LER > 400 lm/W could be achievable at the correlated-colour temperature (CCT) of 4870 K and a rather low CRI < 50, using mixing the light from three LEDs operating at optimized emission wavelengths and having Gaussian emission spectra with a unified width (FWHM) of 30 nm. Increasing the CRI value up to 85 and higher required the use of four LEDs and resulted in the LER lowering below 370 lm/W. CRI > 98 at LER < 330 lm/W was predicted to be attained with five LEDs [1]. After [1], a common opinion has been developed that the best practical compromise between CRI and

LER could be found at the optimal number of individual LEDs (or other light sources) equal four [2-7].

Optimization of the colour mixing of various LED sources was generally carried out under quite different assumptions. In particular, narrow emission lines of ~1–5 nm were considered in [5-7]; the emission spectra 13, 25, and 36 nm wide were attributed to amber/red, blue, and green LEDs in [3]; the spectral widths of 20 and 30 nm were chosen for blue/red and green LEDs in [2], whereas a unified spectral width of 30 nm was assumed for all the LEDs in [1]. On the other hand, the importance of accounting for the actual spectral widths of the individual light sources for correct estimation of potentially achievable CRI and LER was pointed out in [4,8,9].

To our knowledge, the only study [4] considered the real LED efficacy instead of LER and utilized the experimental LED emission spectra and wall-plug efficiency (WPE) as a function of peak wavelength, while optimizing the colour mixing. This study enabled estimations of efficacy and CRI limits for a particular number of cases but did not provide guiding trends or/and optimal solutions for the colour mixing in a wide range of CCT variation. Apparently, this was because of using Monte-Carlo approach to optimization, required a careful post-processing to identify the trends of interest. In addition, the WPE dependence on the peak emission wavelength assumed in [4] has now

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become out of date due to a fast progress in improvement of the LED performance. Therefore, the results reported in [4] should be, in any case, updated with new data on the LED efficiency.

This paper is aimed at finding optimal ways for colour mixing in various white-light sources with regard of their efficacy as the central point of the optimization along with CRI of the emitted light. Two different approaches are compared: (i) that based on mixing of light emitted by individual III-nitride and III-phosphide LEDs only and (ii) that using partial light conversion by a yellow YAG:Ce<sup>3+</sup> phosphor. We content our consideration with a high-quality white-light sources providing CRI close to or greater than ~90 [10]. The maximum achievable efficacy and CRI are estimated for the above approaches and compared with each other using the recently published data on the LED efficiencies and emission spectrum widths.

## 2 Simulation approach

### 2.1 Parameterization of LED characteristics

In order to optimize the colour mixing in a white-light source, one should know how the emission spectrum and WPE of individual LEDs depend on their peak emission wavelength. Following [9], we had collected and approximated the available data on the emission spectra of III-nitride and III-phosphide LEDs [9-13] as a function of the peak emission wavelength (Fig.1a). Then the spectral power density of light emitted by each LED, referred hereafter as the emission spectrum, was approximated with the function

$$I(\lambda) = \frac{A}{\exp[(\lambda - \lambda_m)/\Delta_l] + \exp[(\lambda_m - \lambda)/\Delta_s]} \quad (1)$$

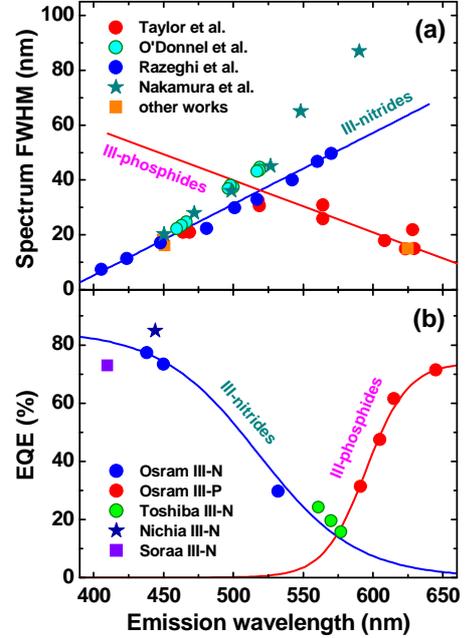
peaking at the wavelength

$$\lambda_p = \lambda_m + \frac{\Delta_l \Delta_s}{\Delta_l + \Delta_s} \ln(\Delta_l / \Delta_s) \quad (2)$$

and having an asymmetric shape relatively to the peak wavelength  $\lambda_p$ . Here,  $A$  was the normalization coefficient providing the integral of the function (1) over the full spectral range to equal unity. The broadening parameters  $\Delta_s$  and  $\Delta_l$  were adjusted in such a way, as to provide (i) FWHM of the emission spectra equal to the experimental one plotted in Fig.1a and (ii) short-wavelength wing of the spectrum corresponding to the sum of the carrier thermal energy (~26 meV at 300 K) and additional uniform broadening of 3 meV found previously from the in-house processing of the data relevant to high-quality blue LEDs. Such an approximation might slightly overestimate the emission spectra asymmetry in the case of III-nitride LEDs at the wavelength greater than ~500 nm. Nevertheless, it accounted properly the existing qualitative difference in the spectrum shapes of nitride and phosphide light emitters.

There was lack of systematic data on the dependence of WPE of various LEDs on their emission wavelength.

Therefore, we employed in simulations the available data on the maximum values of external quantum efficiency (EQE) [14-17] instead of WPE (see Fig.1b). Substitution of WPE by EQE normally provided the inaccuracy of ~1–3% in the WPE value at the peak of its efficiency.



**Figure 1** Spectrum FWHM (a) and maximum EQE (b) of III-nitride and III-phosphide LEDs. Symbols are experimental points borrowed from various literature sources, lines are approximations used in our simulations.

As one can see from Fig.1b, EQE of both III-nitride and III-phosphide LED drops dramatically in the spectral range of ~510-600 nm, thus indicating the well-know 'green-gap' problem. The LEDs emitting in this spectral range have the efficiency lower than ~50%, whereas the maximum efficiencies attained in the case of III-nitride and III-phosphide LEDs are of ~80% and ~70%, respectively. In the simulations, the spectral EQE dependences were approximated by appropriate sigmoid functions (see Fig.1b).

**2.2 Method of optimization** In general,  $N$  peak wavelengths of  $N$  individual LEDs forming the white-light source and  $N - 1$  independent power fractions corresponding to contributions of the LEDs to the total emission spectrum should be found by the colour mixing optimization. In this study, we considered the white light metameric with the radiation of a black-body heated up to the temperature equal to desirable CCT. This constrained the chromatic coordinates  $x$  and  $y$  corresponding to the total emission spectrum of a white-light source examined to belong to the Planckian (black-body radiation) locus in the 1931 CIE 2° chromaticity diagram and to be entirely determined by the CCT value. Such an admission reduced the number of parameters to be varied during the colour mixing optimization to the value of  $2N - 3$  [1]. Following the conventional

approaches to optimization, we neglected possible photon recycling in the LED active regions, considering emission of every LED to be independent of other light emitters.

A special software was developed for the colour mixing optimization, allowing flexible building up multi-parametric objective functions. To find an optimum of the objective function, the Nelder-Mead downhill simplex method [18] was employed. The method was based on a derivative-free algorithm of multi-dimensional optimization. It was fairly efficient to find the optimum in case of a moderate number of the optimization parameters. Being based on direct comparison of the objective function values, the Nelder-Mead method was found to be quite stable with respect to possible inaccuracies in the objective function evaluation.

Typical objective functions used in our study localized CRI at a certain value and enabled finding the maximum value of the efficacy corresponding to the chosen CRI.

**2.3 Colour characteristics of a light source** As soon as the peak emission wavelengths of individual LEDs  $\lambda_k$  ( $k=1 \dots N$ ) and their power fractions  $p_k$  ( $k=1 \dots N$ ) in the total emission spectrum were found by the optimization procedure, the total white-light spectrum could be built-up as the superposition of individual spectra by using the parameterization (1)-(2). In the case of the light conversion by a phosphor, its actual emission spectrum was used instead of the spectral function (1). Then CRI and LER ( $Y_L$ ) corresponding to the white-light spectrum could be found by the conventional way (see, e.g., [1] for more detail).

Finally, the efficacy of the white-light source  $Y$  is calculated as follows:

$$Y = Y_L / \sum_{k=1}^N p_k / \eta_{LED}(\lambda_k) \quad (3)$$

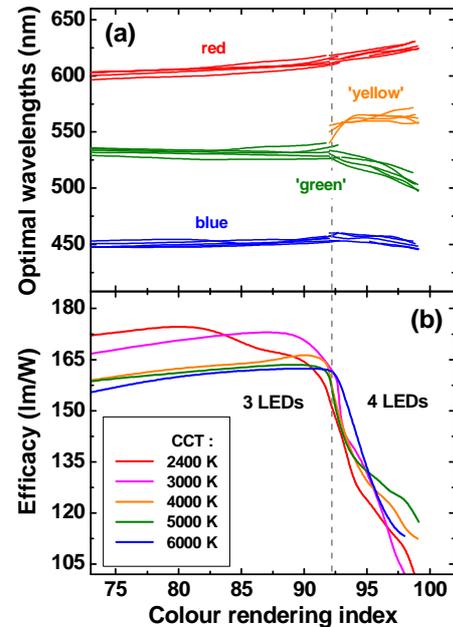
where  $\eta_{LED}$  is the WPE of an individual LED of other light source emitting light at the peak wavelength  $\lambda_k$ .

### 3 Optimization results

**3.1 Light sources comprising of LEDs only** We started the optimization of colour mixing from consideration of four individual LEDs. Those emitting light at the wavelengths shorter than 580 nm were assumed to be made of III-nitride materials, whereas LEDs emitting at longer wavelengths were regarded as III-phosphide devices. Our simulations have revealed a transition from the optimal number of individual LEDs equal four to that equal three, while decreasing the chosen value of CRI. Earlier a similar behaviour was reported in [1]. The critical CRI value corresponding to the transition is found to be 92.2 (see Fig.2).

Figure 2a shows the optimal peak wavelength of LEDs found from the colour mixing optimization (different lines grouped close to each other correspond to different CCT values ranged between 2400 and 6000 K). The optimal wavelengths are found to depend weakly on the chosen

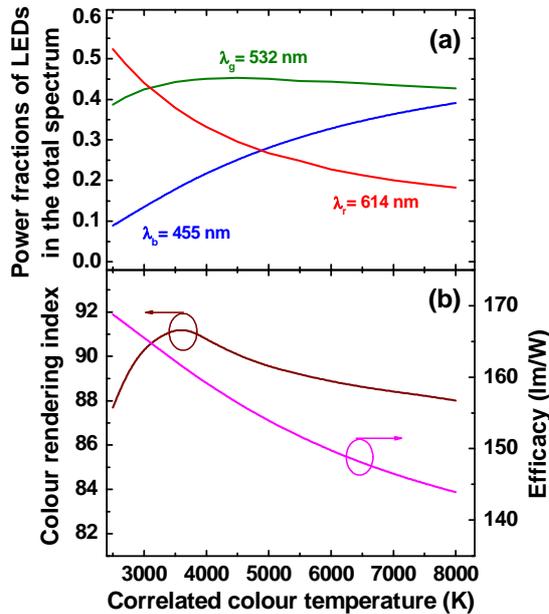
CCT. Their values obtained for various CCTs differ from each other not greater than by  $\sim 10$  nm in the whole range of CRI variation from 75 to 100. The splitting of the optimal emission wavelength in the green spectral range into two branches marked in Fig.2a as 'green' and 'yellow' indicates the above mentioned transition in the optimal numbers of LEDs observed at CRI = 92.2.



**Figure 2** Optimal peak wavelengths (a) and maximum efficacy (b) of white-light LED sources versus CRI calculated for various CCT values. Vertical dashed line separates the regions with the optimal number of individual LEDs equal to three or four.

The optimal peak wavelengths obtained in our simulations for CRI > 92.2 differ noticeably, on average from 7 to 16 nm, from those reported, in [5,6] for the colour mixing of four individual light sources with 1 nm-wide emission lines. Besides, the optimal wavelengths plotted in Fig.2a are more strongly dependent on the CRI value than those recommended in [5,6]. This difference in the predictions may be attributed to the practical widths and asymmetry of the emission spectra accounted for in our simulations.

Figure 2b displays the maximum efficacy of the LED-based white-light sources as a function of CRI computed for various CCTs. The figure demonstrates that the maximum efficacy of  $\sim 165$ -170 lm/W can be achieved at CRI > 90 and CCT = 2400–3000 K by using three individual LEDs. Further increase in CRI can be attained with four LEDs, being accompanied by a dramatic efficacy reduction. This is because two of four LED's emission wavelengths get into the 'green gap' where their WPEs are comparably low, resulting eventually in a lower efficacy. In the case of the colour mixing from three LEDs, however, the emission wavelength of only one of them gets into the 'green gap', which enables achieving a higher efficacy of the resulting white light.



**Figure 3** Power fractions of individual LEDs in the total emission spectra of 'smart' white-light source (a), and CRI/efficacy of the source (b) as a function of CCT.

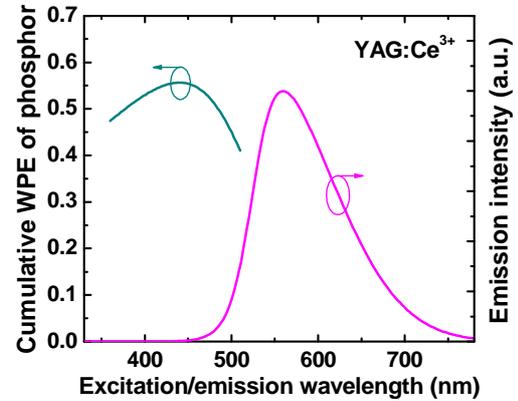
The weak dependence of the optimal emission wavelengths on the CCT value (Fig.2a) points out to feasibility of the development of a 'smart' white-light source with CCT variable by adjusting the power fractions of individual LEDs in the total emission spectrum. Indeed, choosing the emission wavelengths of blue, green, and red LEDs to be 455, 532, and 614 nm, respectively, one can obtain the white light with the rather high and stable CRI ~88–91 and the efficacy of ~145–170 lm/W maintained in a wide range of the CCT variation from 2600 to 8000 K (Fig.3b). For this, adjusting the power fractions of mainly blue and red LEDs would be necessary, whereas that of the green LED could be kept nearly constant in the colour temperature range considered (Fig.3a).

**3.2 Phosphor-converted light sources** White-light sources utilizing partial light conversion by a yellow YAG:Ce<sup>3+</sup> phosphor were considered under the following assumptions. First, the phosphor was regarded as being excited by a blue III-nitride LED only. Second, the absorption of light emitted by the phosphor in a wide spectral range in the active regions of individual LEDs was neglected. In order to regard the phosphor in the manner employed for LEDs, we should specify its emission spectrum and bring into consideration a cumulative WPE of the phosphor  $\eta_{YAG}$  similar to WPE of individual LEDs:

$$\eta_{YAG} = \eta_{LED}(\lambda_b) \eta_i (E_{YAG} / E_{LED}) \quad (4)$$

Here  $\eta_{LED}$  is the WPE of the III-nitride blue LED exciting the YAG phosphor at the wavelength  $\lambda_b$ ,  $\eta_i$  is the quantum yield of the phosphor emission, and  $E_{LED}$  and  $E_{YAG}$  are

the photon energies of the blue LED and the phosphor, respectively, averaged over their emission spectra. The ratio  $E_{YAG}/E_{LED}$  in Eq.(4) accounts for the Stokes shift related to the light down-conversion.



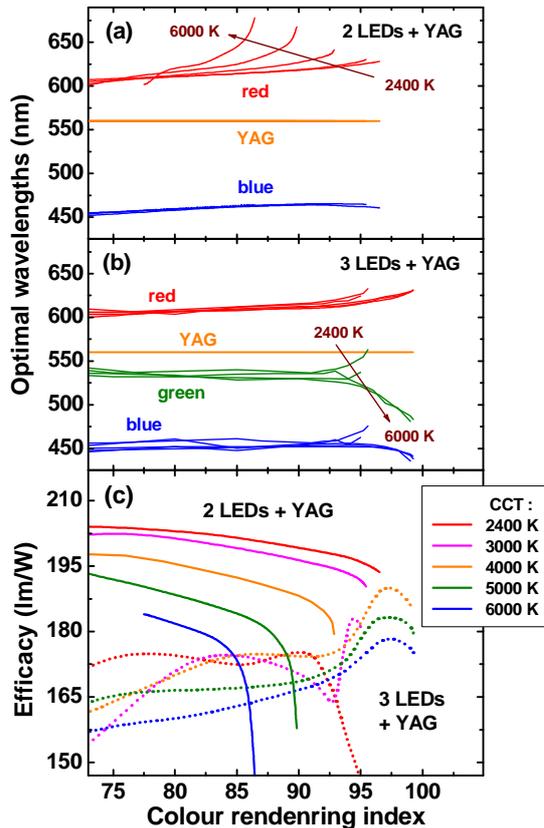
**Figure 4** Emission spectrum (right axis) and cumulative WPE of the YAG:Ce<sup>3+</sup> phosphor excited by a III-nitride LED (left axis) as a function of emission/excitation wavelength, respectively.

Figure 4 displays the emission spectrum of the YAG phosphor obtained by approximation of the data reported in [19–21] and used in our simulations. The spectrum peaks at 560 nm and has FWHM ~120 nm. The cumulative WPE of the phosphor as a function of the excitation wavelength is also shown in Fig.4. It was calculated assuming the quantum yield of the phosphor emission  $\eta_i$  to be ~95% at room temperature. One can see that  $\eta_{YAG}$  has a maximum of ~55.5% at the excitation wavelength of ~440 nm. At longer wavelength, the cumulative WPE decreases because of the LED efficiency reduction close to the 'green gap'. At shorter wavelength, a larger Stokes shift is responsible for the  $\eta_{YAG}$  decrease. We should note that the cumulative WPE of the YAG phosphor is more than two times higher than WPE of a III-nitride LED emitting at 560 nm (compare Fig.4 with Fig.1b). Therefore, we can expect *a priori* the efficacy of the phosphor-converted white-light source to be higher than that of the source comprising of LEDs only. One more conclusion following from Fig.4 is that the use of a violet LED having the emission wavelength of 410 nm for the phosphor excitation [15] does not reduce substantially its cumulative WPE in spite of a larger Stokes shift.

The results of the colour mixing optimization in the white-light source comprising of LEDs and a yellow YAG:Ce<sup>3+</sup> phosphor excited by the blue LED are summarized in Fig.5. In contrast to the case of the white-light sources based on LEDs only, the optimal emission wavelengths of the phosphor-converted light source become dependent on CCT, especially at high CRI values (see Fig.5a and Fig.5b).

Two alternative solutions have been found by optimization for a warm (CCT = 2400–3000 K) and a neutral (CCT = 4000 K) white light. In the case of the warm white

light, CRI ~95–96 at the efficacy of ~190–195 lm/W can be achieved by using two, blue and red, LEDs along with the YAG phosphor (solid lines in Fig.5c). And, the use of three LEDs and the phosphor enables attaining extremely high CRI ~97–98 at the efficacy of ~190 lm/W in the case of the neutral white light (see dotted lines in Fig.5c). Deviation of CCT from 4000 K results in the efficacy reduction.



**Figure 5** Optimal emission wavelengths of white-light sources comprising of two LEDs and phosphor (a) and three LEDs and phosphor (b). The maximum efficacy of the light sources as a function of CRI obtained for the former (solid lines) and latter (dotted lines) cases (c).

The efficacy values of the phosphor-converted white-light source are by ~15% higher than in the case of the light sources comprising of LEDs only. The solution found for the neutral white light turns out to be workable due to the fact that the optimal wavelength of the green LED is shifted from 530 to 485 nm, which is accompanied by almost two-fold increase in the LED efficiency (see Fig.1b) and, eventually, by the efficacy rise.

**3.3 Comparison of the approaches** Comparison of the maximum achievable CRI and efficacy obtained for two types of the white-light source considered shows that the approach based on the partial light conversion by a yellow YAG:Ce<sup>3+</sup> phosphor is more advantageous. A higher CRI achievable in this case is due to a much wider emission spectrum of the phosphor compared to that of the

LED having the same peak emission wavelength. This conclusion is in line with that made from an analysis of superior colour rendition in white-light sources [8]. The phosphor-converted white-light sources provide also a systematically higher efficacy. The reason for that is a remarkably higher cumulative WPE of the YAG phosphor, as compared to WPE of the LED emitting at the same or close wavelength.

The yellow phosphor considered in our study is not, however, optimal for maximizing the efficacy of the phosphor-converted light source. First, it has the extended long-wavelength wing spread up to and beyond the upper border of the visual spectrum, i.e. 780 nm (Fig.4), which reduces the efficacy of the white-light source [22]. Therefore, more narrow phosphor emission spectrum would be advantageous over that produced by the YAG:Ce<sup>3+</sup> one for the efficacy increase. Second, a shift of the phosphor emission spectrum from the yellow to green spectral range is also expected to improve the efficacy, being better adopted to the maximum sensitivity of the photopic vision. Just these improvements underlie the 'brilliant mix' concept of the white-light sources [23] suggested three years ago for simultaneous increase of their CRI and efficacy.

**4 Summary** In this paper, we have considered optimal ways for colour mixing in the white-light sources either comprising of LEDs only or utilizing partial light conversion by a yellow YAG:Ce<sup>3+</sup> phosphor. The maximum achievable CRI and efficacy are estimated for the above two cases, using the most recent data on the LED efficiency as a function of the emission wavelength.

In the case of the light sources comprised of LEDs only, the maximum efficacy of ~165–170 lm/W can be achieved at CRI ~ 90–92 and CCT = 2400–3000 K by using three individual LEDs. Further improvement of CRI requires the use of four LEDs and is accompanied by a dramatic efficacy reduction originated from the 'green gap' effect on the LED efficiencies. On the other hand, such a type of the light source is quite suitable for producing high-quality 'smart' white light with CCT variable by adjusting the contributions of individual LEDs to the total emission spectrum. Our simulations predict the 'smart' white-light source to be capable of providing stable colour rendition (CRI = 88–91) in the wide, 2600–8000 K, range of the CCT variation.

Two different optimal solutions have been found in the case of phosphor-converted white-light source, utilizing a yellow YAG:Ce<sup>3+</sup> phosphor pumped with a blue LED. One is based on the use of two, blue and red, LEDs with the phosphor and is advantageous for producing a warm (CCT = 2400–3000 K) white light. Here, the maximum efficacy of ~190–195 lm/W can be achieved at CRI ~95–96. The workability of the above approach is confirmed by the practical implementation of the 'brilliant mix' concept suggested previously. Another solution applicable for the neutral (CCT = 4000 K) white light employs three LEDs oper-

ating with the YAG phosphor and provides the maximum efficacy of  $\sim 190$  lm/W at excellent colour rendition (CRI  $\sim 97$ – $98$ ).

Comparison of the approach utilizing the partial light conversion by a phosphor with that using the LED emission only demonstrates that the former one is more advantageous in terms of both CRI and efficacy. In our opinion, such a rank will be kept unchanged until the efficiency of LEDs in the 'green gap' is improved substantially.

The above results have been obtained with the reference to the maximum LED efficiencies reported to date. These efficiencies can be normally obtained at the operating currents of LEDs that are too low for practical applications. Increasing the currents reduces the LED efficiencies because of the well-known droop effect [24]. Our preliminary simulations made for the practical operating currents predict the efficacy reduction by  $\sim 25$ – $30\%$ , as compared to the maximum achievable values.

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